

Note: We will usually express probabilities in reduced fraction form. Decimals are also allowed. Sometimes probabilities are also expressed as percentages, although this is discouraged in general.

Definition: Let E be an event (i.e. a set of outcomes). The *complement* of E , denoted by \bar{E} , is the set of all outcomes not included in E .

Example 4: Find the complement of each event:

- (a) Choosing a fruit from a bowl of apples, oranges and bananas and getting a banana.
- (b) Rolling a die and getting an odd number.
- (c) Drawing a card and getting a face card.

(a) Getting an apple or an orange. (b) Getting an even number. (c) Getting an ace or a numbered card.

Facts about Probabilities:

- (1) For any event E , $0 \leq P(E) \leq 1$.
- (2) If an event is impossible (i.e. there are no outcomes in E), then $P(E) = 0$.
- (3) If an event is certain (i.e. every outcome is in E), then $P(E) = 1$.
- (4) The sum of the probabilities for all outcomes in the sample space is 1.
- (5) $P(\bar{E}) = 1 - P(E)$.

Example 5: If the probability that a person owns a mp3 player is $\frac{3}{10}$, find the probability that a person does not own a mp3 player.

Since owning an mp3 player and not owning an mp3 player are complementary events,

$$\begin{aligned} P(\text{not owning an mp3 player}) &= 1 - P(\text{owning an mp3 player}) \\ &= 1 - \frac{3}{10} \\ &= \frac{7}{10} \end{aligned}$$

Empirical Probability

In empirical probability, the different outcomes are not necessarily equally likely. Suppose we perform an experiment (hence “empirical”) and flip a coin

1000 times. Classical probability says that we should get 500 heads and 500 tails, but that is very unlikely to work out *exactly* that way. Our actual results could be summarized by a frequency distribution.

Class	Frequency
Head	523
Tail	<u>477</u>
	1000

So what is the probability that a randomly selected flip will be a head? It's $\frac{523}{1000}$. So empirical probability is based on observations.

Example 6: For a recent year, 51% of families in the United States had no children under the age of 18; 20% had one such child; 19% had two children; 7% had three children; and 3% had four or more children. If a family is selected at random, find the probability that the family has:

- (a) Two or three children.
- (b) More than one child.
- (c) Less than four children.

(a) .26 (b) .29 (c) .97

Example 7: Suppose we are drawing a card from a standard deck of cards. What are the following probabilities?

- (a) Getting a 5 or a jack.
- (b) Getting a heart or spade.
- (c) Getting a 10 or a diamond.

(a) There are 8 cards that are 5's or jacks, so the probability is

$$P(5 \text{ or } J) = \frac{8}{52} = \frac{2}{13}.$$

(b) There are 26 such cards, so $P(H \text{ or } S) = \frac{26}{52} = \frac{1}{2}$.

(c) There are 16 such cards, so $P(10 \text{ or } D) = \frac{16}{52} = \frac{4}{13}$.

Are parts (a) and (b), we just added the two individual probabilities together, but on part (c) that didn't work. Why not?

Definition: Two events are *mutually exclusive* if they cannot occur at the same time.

Addition Rule For Probabilities

1. If A and B are mutually exclusive events, then

$$P(A \text{ or } B) = P(A) + P(B).$$

2. If A and B are *not* mutually exclusive events, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

Example 8: In a statistics class with 24 students, there are 11 seniors and 13 juniors. Nine of the seniors are female and 7 of the juniors are male. If a student is selected at random, find the probability that the selected student is: (a) a junior or a female, (b) a senior or a female, (c) a junior or a senior.

$$(a) P(J \text{ or } F) = P(J) + P(F) - P(J \text{ and } F)$$

$$= \frac{13}{24} + \frac{15}{24} - \frac{6}{24}$$

$$= \frac{22}{24} = \frac{11}{12}$$

$$(b) P(S \text{ or } F) = P(S) + P(F) - P(S \text{ and } F)$$

$$= \frac{11}{24} + \frac{15}{24} - \frac{9}{24}$$

$$= \frac{17}{24}$$

$$(c) P(J \text{ or } S) = P(J) + P(S)$$

$$= \frac{13}{24} + \frac{11}{24}$$

$$= \frac{24}{24} = 1$$